

Reg. No. :

K22U 0413

Name :

VI Semester B.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2022
(2019 Admission)
CORE COURSE IN MATHEMATICS
6B10 MAT : Real Analysis – II

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

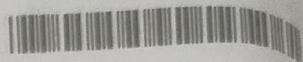
1. Show that $f(x) = x^2$ defined on $[0, 3]$ is uniformly continuous.
2. Define Riemann integral of a function f over $[a, b]$.
3. Test the convergence of the integral $\int_0^1 \frac{1}{1-x} dx$.
4. Show that $\beta(m, n) = \beta(n, m)$.
5. Define a metric on a set S .

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Let $f : A \rightarrow \mathbb{R}$ is a Lipschitz function. Show that f is uniformly continuous on A .
7. If f and g are increasing functions on A , then show that $f + g$ is an increasing function on A .
8. If $f \in \mathcal{R}[a, b]$, then prove that f is bounded on $[a, b]$.
9. Show that every constant function on $[a, b]$ is Riemann integrable.
10. If $f, g \in \mathcal{R}[a, b]$, and $f(x) \leq g(x)$ for all $x \in [a, b]$, then prove that $\int_a^b f \leq \int_a^b g$.

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11. Evaluate $\int_0^3 \frac{1}{(x-1)^{3/2}} dx$.

12. Test the convergence of the integral $\int_1^{\infty} \frac{\sin^2 x}{x^2} dx$.

13. Show that $\beta(p, q) = \int_0^{\pi/2} \sin^{2p-1} \theta \cos^{2q-1} \theta d\theta$.

14. Let S be a nonempty set. For $s, t \in S$, define

$$d(s, t) = \begin{cases} 1 & \text{if } s \neq t \\ 0 & \text{if } s = t \end{cases}$$

Show that d is a metric on S .

15. Let $f_n(x) = x + \frac{1}{n}$ for $x \in \mathbb{R}$ and $n \in \mathbb{N}$. Show that f_n converges to $f(x) = x$ uniformly on $\mathbb{R}$.

16. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n!} x^n$.

PART - C

Answer **any four** questions. **Each** question carries **four** marks.

17. Let I be a closed bounded interval and let $f : I \rightarrow \mathbb{R}$ be continuous on I , then show that f is uniformly continuous on I .

18. Suppose $f : [a, b] \rightarrow \mathbb{R}$ is continuous on $[a, b]$. Show that $f \in \mathcal{R}[a, b]$.

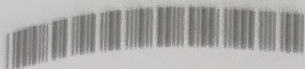
19. Suppose $g \in \mathcal{R}[a, b]$ and $f(x) = g(x)$ except for a finite number of points on $[a, b]$. Show that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \int_a^b g$.

20. Show that $\Gamma n \cdot \Gamma(1-n) = \frac{\pi}{\sin n\pi}$.

21. Evaluate $\Gamma\left(\frac{1}{2}\right)$ and $\Gamma\left(-\frac{1}{2}\right)$.

22. Evaluate $\int_{-1}^1 \frac{dx}{x^{2/3}}$.

23. Let $f_n(x) = x^n$ for $x \in [0, 1]$ and $n \in \mathbb{N}$. Find a function $g(x)$ in $[0, 1]$ such that f_n converges to g pointwise on $[0, 1]$.



PART - D

Answer **any two** questions. Each question carries **6** marks.

24. Let $I \subset \mathbb{R}$ be an interval, $f : I \rightarrow \mathbb{R}$ be strictly monotone and continuous on I . Show that the function g inverse to f is strictly monotone and continuous on $J = f(I)$.

25. State and prove Cauchy Criterion for Riemann integrability.

26. a) Show that $\int_0^{\infty} \frac{1}{x^p} dx = \begin{cases} \frac{1}{p-1} & \text{if } p > 1 \\ \infty & \text{if } p < 1 \end{cases}$

b) Show that $\beta(m, n) = \frac{\Gamma m \cdot \Gamma n}{\Gamma(m+n)}$.

27. Let f_n be a sequence of functions in $\mathcal{R}[a, b]$ and f_n converges uniformly on $[a, b]$ to f . Show that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n \rightarrow \infty} \int_a^b f_n$.

PART - E

Answer **any 4** questions. Each question carries **20** marks.

- Find real part and imaginary part of $f(z) = \frac{1}{z}$ at $z = i$.
- Check whether $f(z) = \cos \bar{z}$ is analytic or not.
- Define an entire function and write examples of an entire function.
- Evaluate $\int_D \operatorname{Re} z \, dz$, where D is the quarter disc from $1 + i$ to $3 + 3i$.